Cell Stay Time Analysis under Random Way Point Mobility Model in WLAN

Floriano De Rango, *Member, IEEE,* Peppino Fazio, *Student Member, IEEE,* and Salvatore Marano, *Member, IEEE*

***Abstract*— This letter presents an analytical framework of the Cell Stay Time (CST) in wireless LAN environment. CST is an important parameter that can be used to estimate how long a mobile user will stay in a particular cell and how many cells he will probably visit during his call holding time. CST can be also used to make predictive advanced reservations. A probability density function p.d.f. of the CST is obtained and an analytical expression of its main parameters such as the average and standard deviation values is outlined as a function of the average mobile user speed and cell radius.**

***Index Terms*— Cell stay time, WLAN, random way point mobility model.**

I. Introduction

THIS letter describes an analytical framework to evaluate the cell residential time called Cell Stay Time (CST) of a mobile user in a Wireless LAN (WLAN) coverage area. The cell residential time is an important parameter in wireless networks because it permits the evaluation of how long a user will stay in a cell during its call holding time and how many cells he will visit. This can be useful in resource reservations for an environment supporting node mobility [1]. In previous works, an idea of CST usage for reservations in wireless networks is shown [2]. In these works an early evaluation of CST under different speeds was considered and a prediction of the number of cells visited by mobile nodes was obtained [3]. However, no mathematical formula has been obtained to calculate the CST through the knowledge of network parameters such as mobile user speed and cell radius. In this work a further contribution is made to CST evaluation through an explicit math formula that binds the average speed of the mobile user, the variance around the average speed and the cell diameter to the CST estimation. This letter is organized as follows: section II provides an analysis by simulation of the CST in order to obtain a probability density function p.d.f.; a polynomial regression is considered in section III in order to bind the mean μ and the standard deviation σ of the CST with the average speed, the variation α of the mobile hosts and the cell radius; section IV carries out a performance evaluation of the proposed model; finally, conclusions are summarized in section V.

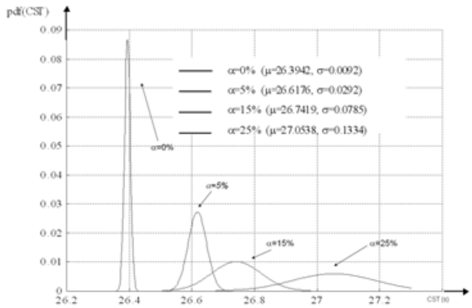

Fig. 1. An example of CST Gaussian distribution and its approximation for $\bar{\nu} = 15Km/h$ and $\alpha = 0$.

II. Cell Stay Time Analysis

In our work the Random Way Point model (RWPM) was considered [4] as a method to describe users' mobility and, first of all, many simulations were carried out in order to evaluate the average CST, with an average speed ν and a variation coefficient α; in this way, the considered users' speed is uniformly distributed in the interval $[\nu - \alpha; \nu + \alpha]$. After a results analysis, a CST distribution was obtained like the one depicted in Fig. 1, with a Gaussian approximation for fixed values of speed and variation. So, the general expression of the CST p.d.f. is:

$$f_{X_{CST}}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad (1)$$

where $\mu = \mu_{CST}(\nu, \alpha, R)$ and $\sigma = \sigma_{CST}(\nu, \alpha, R)$ are respectively the average and standard deviation of the Gaussian distribution. R represents the cell radius. In the next section, analytical expressions for μ and σ are derived with the application of the regression theory.

Fig. 1 shows how the CST distribution parameters change for different α values; it can be observed that both μ and σ increase for higher values of α, because of the higher fluctuations of chosen speeds during hosts movements. The Kolmogorov-Smirnov (KS) test [6] has been employed to evaluate the correctness of a Gaussian approximation of the CST distributions under the RWPM; table I summarizes the obtained p-values for different values of α, with *D*=300 and ν=15Km/h.

The cumulative *distribution function* c.d.f. of the CST from (1) is:

$$F(x) = P(X_{CST} \le x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} dt = 1 - \epsilon \qquad (2)$$

and through (2), the probability that CST is lower than a value x with a fixed error threshold ϵ is obtained such as referred in

TABLE I

VALUES OF μ AND σ OF CST DISTRIBUTIONS AND KS P-VALUES FOR DIFFERENT MOBILITY PARAMETERS.

α	μ_{CST}	σ_{CST}	KS p-value
0	68.39245	0.055022	0.5305
5	70.76407	0.128481	0.5087
15	70.9836	0.568966	0.6712
25	71.3662	0.829614	0.7012

[5]. Thus the knowledge of μ_{CST} and σ_{CST} is necessary to obtain a good estimation of CST, depending on ν, α and R.

III. CST POLYNOMIAL REGRESSION IN THE 3-D SPACE

The regression analysis was performed under MATLAB application and the minimum observed value of the *determination coefficient* R^2 over all obtained polynomial functions is 0.9898. The curves of the average μ_{CST} and σ_{CST} for different system parameters values are shown in Fig. 2. Details on polynomial regression technique can be found in [6]. Fig. 2a shows the relationship between μ_{CST}, the users' average speed ν and system cells' diameter *D*: maintaining *D* at a constant value, the curve has a decreasing course for increasing values of ν, because of the lower duration of the users' average permanence in a cell; moreover, as can be expected, fixing a value for the users' average speed *v*, the average CST μ_{CST} always increases, for higher values of the cell dimension; that is, a mobile host takes more time to fully cross a coverage area. In Fig. 2b, fixing *D*, the σ_{CST} increases for higher *D* values. Fig. 2c shows the relationship between μ_{CST}, the users' average speed *v* and its variation coefficient α: maintaining α at a constant value, the curve decreases for increasing values of ν, because of the same reason explained previously; in addition, fixing a value for users' average speed ν, the average CST μ_{CST} increases slightly, for higher values of α; that is because the chosen speed values can vary in a larger range and the CST random variable has a higher standard deviation σ_{CST}. On the other hand, Fig. 2d shows that σ also increases for higher values of α, because the higher probability of a speed change.

The general equation of the average of the CST is expressed in (4) with a fourth order polynomial regression:

$$\mu_{CST}(\bar{\nu}) = n_4\bar{\nu}^4 + n_3\bar{\nu}^3 + n_2\bar{\nu}^2 + n_1\bar{\nu} + n_0 \quad (3)$$

where $n_i = f(D, \alpha)$ with i=0, 1, ..,5 and *D=2R*. Because the n_i terms follow a linear dependence on *D*, they can be expressed as:

$$n_i = a_i D + b_i \quad (4)$$

μ_{CST} can be represented in the following way:

$$\mu_{CST}(\bar{\nu}) = [n_0, n_1, ..., n_4,] \cdot \left[1, \bar{\nu}, \bar{\nu}^2, \bar{\nu}^3, \bar{\nu}^4\right]^T = \langle n \rangle^T \cdot \langle \bar{\nu} \rangle^{n=4} \quad (5)$$

where the notation $\langle\cdot\rangle$ is used to represent a column vector and $\langle\cdot\rangle^T$ is the transpose operator applied to the vector. In (6) $\langle\nu\rangle^i = \left[1, \bar{\nu}, \ldots, \ldots, \bar{\nu}^i\right]$ is a $(i+1)x1$ vector. In order to calculate the coefficient n_i it is important to evaluate coefficients a_i and b_i. After a second order regression between a_i and α, the following expression can be obtained:

$$a_i = m_{i1}\alpha^2 + m_{i2}\alpha + m_{i3} \quad (6)$$

with i= $0, \ldots, 5$.

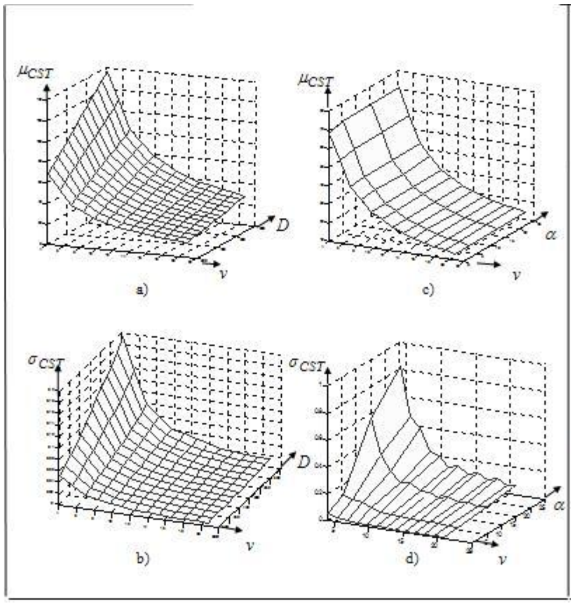

Fig. 2. Matlab Graphics: a) and c) 3-D plot of μ_{CST}; b) and d) 3-D plot of σ_{CST}

Considering another polynomial regression analysis (5-th order in this case) on the b_i coefficients for different α values around the average speed of the mobile nodes, the expression of the b_i terms can be represented as follows:

$$b_i = c_{i1}\alpha^5 + c_{i2}\alpha^4 + c_{i3}\alpha^3 + c_{i4}\alpha^2 + c_{i5}\alpha + c_{i6} \quad (7)$$

with i= $0, \ldots, 5$.

Using a matrix notation, the terms a_i can be calculated as follows:

$$A = M \cdot \langle\alpha\rangle^{n=2} \quad (8)$$

where A is a (5x1) vector, M is a (5x3) matrix and $\langle\alpha\rangle^{n=2}$ is a (3x1) vector. Terms b_i, instead, are represented in the following form:

$$B = C \cdot \langle\alpha\rangle^{n=5} \quad (9)$$

where B is a (5x1) vector, C is a (5x6) matrix and $\langle\alpha\rangle^{n=5}$ is a (6x1) vector. Thus, the final matrix expression of the CST is the following:

$$A = \left[\left(M \cdot \langle\alpha\rangle^{n=2}\right) \cdot D + C \cdot \langle\alpha\rangle^{n=5}\right] \cdot \langle\nu\rangle^{n=4} \quad (10)$$

The coefficients of the matrixes *M* and *C* are expressed in Fig. 3 and Fig. 4. The same regression analysis was carried out for the standard deviation course, on varying mobility parameter α.

A first polynomial regression of 4th order of σ as function of ν has been obtained:

$$\sigma_{CST}(\bar{\nu}) = m_4\bar{\nu}^4 + m_3\bar{\nu}^3 + m_2\bar{\nu}^2 + m\bar{\nu} + m_0 \quad (11)$$

Thus:

$$\sigma_{CST}(\bar{\nu}) = [m_0, m_1, \ldots, m_4] \cdot [1, \bar{\nu}, \ldots, \bar{\nu}^4]^T = \langle m \rangle^T \cdot \langle \bar{\nu} \rangle^{n=4} \quad (12)$$

Matrix *M'* in Fig. 6 summarizes m_i values (on the columns) for different *D* values (on the rows) with i=0, 1, ..., 4. A 2nd regression of 5th order for different D values for each m_i term with i=1,2,..4 has been performed:

$$m_i(D) = \beta_{i5}D^5 + \beta_{i4}D^4 + \beta_{i3}D^3 + \beta_{i2}D^2 + \beta_{i1}D + \beta_{i0} \quad (13)$$

$$M = \begin{pmatrix} 2,4\cdot 10^{-9} & 5,1\cdot 10^{-8} & 5,4\cdot 10^{-6} \\ 1,5\cdot 10^{-7} & -3,2\cdot 10^{-7} & -3,5\cdot 10^{-4} \\ -3,3\cdot 10^{-6} & 7,4\cdot 10^{-5} & 8,5\cdot 10^{-3} \\ 3,3\cdot 10^{-5} & -7,6\cdot 10^{-4} & -9,7\cdot 10^{-2} \\ -1,2\cdot 10^{-4} & 3,1\cdot 10^{-3} & 5,1\cdot 10^{-1} \end{pmatrix}$$

Fig. 3. Matrix M

$$C = \begin{pmatrix} 4,1\cdot 10^{-10} & -2,2\cdot 10^{-8} & 3,3\cdot 10^{-7} & -7.2\cdot 10^{-7} & -3.7\cdot 10^{-6} & -3.4\cdot 10^{-6} \\ -2,2\cdot 10^{-8} & 1,2\cdot 10^{-5} & -1,8\cdot 10^{-5} & 3.5\cdot 10^{-5} & 2.0\cdot 10^{-4} & 2.4\cdot 10^{-4} \\ 4,1\cdot 10^{-7} & -2,2\cdot 10^{-5} & 3,3\cdot 10^{-4} & -5.9\cdot 10^{-4} & -3.5\cdot 10^{-3} & -6.4\cdot 10^{-3} \\ -3,3\cdot 10^{-6} & 1,8\cdot 10^{-4} & -2,6\cdot 10^{-3} & 4.3\cdot 10^{-3} & 2.4\cdot 10^{-2} & 7.9\cdot 10^{-2} \\ 1,0\cdot 10^{-5} & -5,6\cdot 10^{-4} & 8,5\cdot 10^{-3} & -1.8\cdot 10^{-2} & -3.2\cdot 10^{-2} & -3.9\cdot 10^{-1} \end{pmatrix}$$

Fig. 4. Matrix C

$$\beta = \begin{pmatrix} 1.09\cdot 10^{-16} & -2.46\cdot 10^{-13} & 2.18\cdot 10^{-10} & -9.52\cdot 10^{-8} & 2.04\cdot 10^{-5} & -1.72\cdot 10^{-3} \\ -5.81\cdot 10^{-15} & 1.30\cdot 10^{-11} & -1.15\cdot 10^{-8} & 5.02\cdot 10^{-6} & -1.07\cdot 10^{-3} & 9.08\cdot 10^{-2} \\ 1.086\cdot 10^{-13} & 2.43\cdot 10^{-10} & 2.14\cdot 10^{-7} & -9.32\cdot 10^{-5} & 1.99\cdot 10^{-2} & -1.68\cdot 10^{0} \\ -8.41\cdot 10^{-13} & 1.87\cdot 10^{-9} & -1.65\cdot 10^{-6} & 7.14\cdot 10^{-4} & -1.52\cdot 10^{-1} & 12.8\cdot 10^{0} \\ 2.28\cdot 10^{-12} & -5.06\cdot 10^{-9} & 4.43\cdot 10^{-6} & -1.90\cdot 10^{-3} & 4.04\cdot 10^{-1} & -33.8\cdot 10^{0} \end{pmatrix}$$

Fig. 5. Matrix β

$$M' = \begin{pmatrix} 2.20\cdot 10^{-6} & -1.36\cdot 10^{-4} & 3.15\cdot 10^{-3} & -3.32\cdot 10^{-2} & 1.44\cdot 10^{-1} \\ 2.91\cdot 10^{-6} & -1.78\cdot 10^{-4} & 4.08\cdot 10^{-3} & -4.23\cdot 10^{-2} & 1.78\cdot 10^{-1} \\ 3.61\cdot 10^{-6} & -2.21\cdot 10^{-4} & 5.02\cdot 10^{-3} & -5.14\cdot 10^{-2} & 2.13\cdot 10^{-1} \\ 3.62\cdot 10^{-6} & -2.25\cdot 10^{-4} & 5.23\cdot 10^{-3} & -5.47\cdot 10^{-2} & 2.32\cdot 10^{-1} \\ 3.63\cdot 10^{-6} & -2.29\cdot 10^{-4} & 5.43\cdot 10^{-3} & -5.80\cdot 10^{-2} & 2.52\cdot 10^{-1} \\ 4.39\cdot 10^{-6} & -2.78\cdot 10^{-4} & 6.57\cdot 10^{-3} & -6.98\cdot 10^{-2} & 2.98\cdot 10^{-1} \\ 5.15\cdot 10^{-6} & -3.26\cdot 10^{-4} & 7.72\cdot 10^{-3} & -8.16\cdot 10^{-2} & 3.45\cdot 10^{-1} \\ 6.55\cdot 10^{-6} & -4.02\cdot 10^{-4} & 9.18\cdot 10^{-3} & -9.40\cdot 10^{-2} & 3.87\cdot 10^{-1} \\ 7.94\cdot 10^{-6} & -4.77\cdot 10^{-4} & 1.06\cdot 10^{-2} & -1.06\cdot 10^{-1} & 4.29\cdot 10^{-1} \\ 7.53\cdot 10^{-6} & -4.61\cdot 10^{-4} & 1.05\cdot 10^{-2} & -1.08\cdot 10^{-1} & 4.48\cdot 10^{-1} \\ 7.11\cdot 10^{-6} & -4.46\cdot 10^{-4} & 1.04\cdot 10^{-2} & -1.10\cdot 10^{-1} & 4.66\cdot 10^{-1} \\ 7.94\cdot 10^{-6} & -4.98\cdot 10^{-4} & 1.16\cdot 10^{-2} & -1.23\cdot 10^{-1} & 5.19\cdot 10^{-1} \\ 8.77\cdot 10^{-6} & -5.51\cdot 10^{-4} & 1.29\cdot 10^{-2} & -1.35\cdot 10^{-1} & 5.71\cdot 10^{-1} \end{pmatrix}$$

Fig. 6. Matrix M'

where β_{ij} is the polynomial coefficient with j=0..5 and i=0..4.

$$m_i(D) = [\beta_{i0}, \ldots, \beta_{i5}] \cdot [1, D, D^2, D^3, D^4, D^5]^T = \langle \beta_i \rangle^T \cdot \langle D \rangle^{n=5} \quad (14)$$

A third regression of 4th order is associated to the α variable such shown below:

$$\beta_{ij}(\alpha) = \gamma_{ij4}\alpha^4 + \gamma_{ij3}\alpha^3 + \gamma_{ij2}\alpha^2 + \gamma_{ij1}\alpha + \gamma_{ij0} \quad (15)$$

Thus β is a 5x6 matrix (as depicted in Fig. 5), where each column is given by the following product:

$$\beta_i = [\gamma_{ij}] \cdot [\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4,]^T \quad (16)$$

IV. Simulation Results

In order to appreciate the accuracy of the proposed model, some simulation campaigns were carried out; our scenario consists of 10 wireless cells (each one covered by an access point) with a radius length changing in the range [150-300] meters; mobile hosts move inside the coverage areas in a circular way (a user who is handed-out from the 10-th cell will hand-in the first cell), following a 1-D RWPM [3]; the proposed model is employed in our WLAN simulated net, in order to predict the number of cells that mobile host will probably visit (an important issue for prediction purposes). An example of a possible application of CST prediction is the reservation of bandwidth among cells that the mobile users will probably visit during the call holding time (CHT). Fig. 7 shows the ratio in percentage value between the number of users that do not find the available resources and the number of total users that move among the WLAN cells (extra percentage error); the curves show that the percentage error is under 7%,

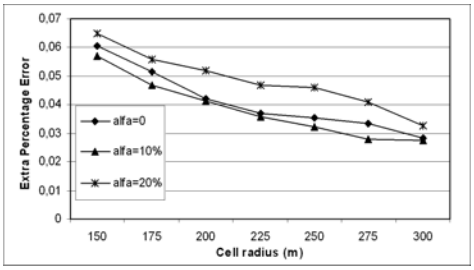

Fig. 7. Percentage of number of visited cell in comparison with predicted cells.

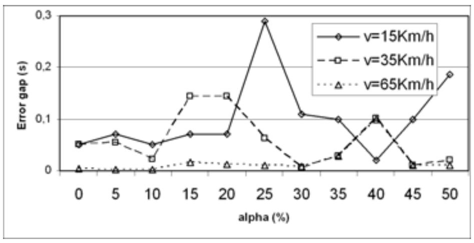

Fig. 8. Gap vs α percentage for different average ν values.

confirming that the estimated CST represents a good way to obtain the number of predicted cells during the CHT.

Fig. 8 shows the average time-gap (s) between the predicted hand-off times and the real hand-off times for different α and ν values: as it can be expected, there are no important differences with the samples-based case if the obtained regression function is adopted for prediction purposes. In particular it is possible to see as the maximum time-gap is below 0.3s.

V. Conclusions

An analytical study of the CST in WLAN networks has been proposed. The CST has been evaluated as a function of the average speed of the mobile user and its variation around the average speed. A dependence of CST on the cell radius (diameter) has also been considered and a regression analysis has been performed in order to calculate an analytical expression for the CST. Simulation results show the expression of CST distribution parameters as a function of percentage variation around the average speed and the cell diameter D permits prediction of the right number of visited cells with a percentage error lower than 7%.

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